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III—A Stochastic Rain Fade Control Algorithm for Satellite Link Power via Nonlinear Markov Filtering Theory

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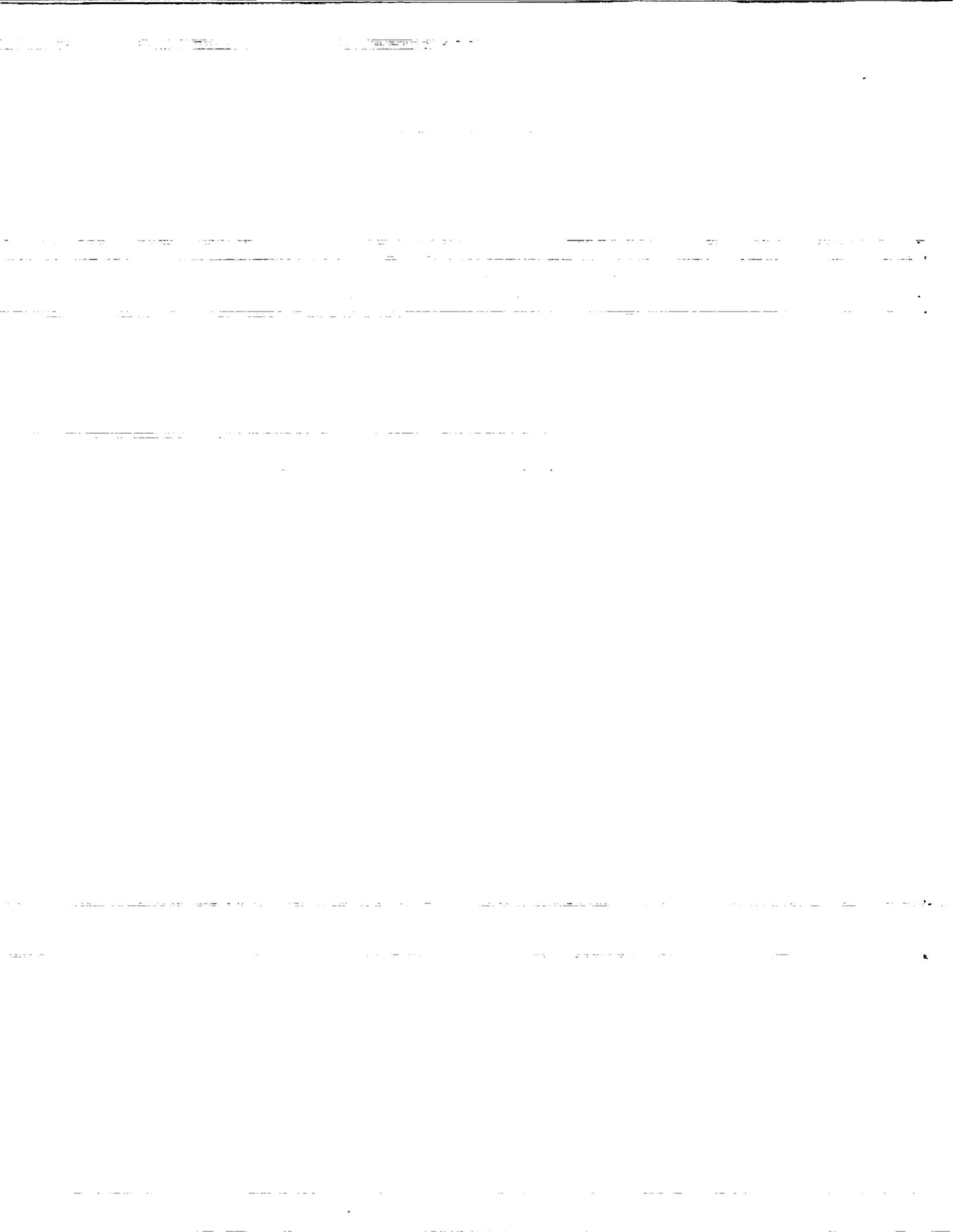
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A STATISTICAL RAIN ATTENUATION PREDICTION MODEL WITH APPLICATION TO THE ADVANCED COMMUNICATION TECHNOLOGY SATELLITE PROJECT

III- A Stochastic Rain Fade Control Algorithm for Satellite Link Power Via Nonlinear Markov Filtering Theory

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SUMMARY

The dynamic and composite nature of propagation impairments that are incurred on earth-space communications links at frequencies in and above the 30/20 GHz Ka band, i.e., rain attenuation, cloud and/or clear air scintillation, etc., combined with the need to counter such degradations after the small link margins (typical of such frequency bands) have been exceeded, necessitate the use of dynamic statistical identification and prediction processing of the fading signal in order to optimally estimate and predict the levels of each of the deleterious attenuation components. Such requirements are being met in NASA's Advanced Communications Technology Satellite (ACTS) Project by the implementation of optimal processing schemes derived through the use of the ACTS Rain Attenuation Prediction Model and nonlinear Markov filtering theory. The ACTS Rain Attenuation Prediction Model discerns climatological variations on the order of 0.5° in latitude and longitude in the continental U.S. The static portion of the model gives precise availability predictions for the "spot beam" links of ACTS. However, the structure of the dynamic portion of the model, which yields performance parameters such as fade duration probabilities, etc., is isomorphic to the state-variable approach of stochastic control theory and, as shown in this paper, is amenable to the design of such statistical fade processing schemes which can be made specific to the particular climatological location at which they are employed.

I. INTRODUCTION

The development of the ACTS Rain Attenuation Prediction Model [1,2,3], another in a variety of such models that now exists, was in response to several system design and performance requirements of satellite systems in and above 30/20 GHz (the Ka band); Such a satellite system is NASA's Advanced Communication Technology Satellite (ACTS) Project after which the model is named. Two major requirements are the ability to 1) estimate the level of a communication signal fade due to rain on a satellite link using, in the general case, attenuation measurements that may be corrupted with random as well as systematic measurement errors (defined here as *identification*) and 2) predict what such attenuation levels will prevail a short time into the future so as to forewarn the need for the deployment of a fade countermeasure (defined here as *prediction*). For example, a situation may exist for the user of a small remote satellite terminal to employ the received communications channel, with all its attendant power fluctuations due to modulation, etc., as the source of measurement of link attenuation due to rain so as to drive some fade mitigation technique, the use of which is needed beyond some pre-established fade threshold. Or, as in the case of ACTS, one may be operating at a frequency that is not only impaired by rain but also by the phenomena of clear-air and/or cloud scintillation; here, one receives the total fading signal and, if one is to have a reliable satellite communications link, must separate out the component rain and scintillation effects since each must be dealt with in a different manner (e.g., rain fade by power control and scintillation, if it proves to be a problem, by time diversity transmission).

It is the purpose of this paper to indicate how results of the dynamic portion of the ACTS Rain Prediction Model can be used in the implementation of processing schemes that are robust to such scenarios indicated above. In particular, the simplest case is considered whereby one has available attenuation measurements with associated measurement errors at discrete points in time. From these discrete, "noisy" observations of link attenuation, it is required to obtain optimal estimates of not only the satellite link attenuation that corre-

sponded to the measurement, but also an optimal prediction of what the attenuation value will be at the next (future) sampling time. This problem is formed within the context of non-linear Markov filtering. The optimality criterion used will be the minimization of the least square error that exists between the predicted value of attenuation and that estimated from the noisy measurement process. The only fading mechanism that will be considered here is that due to rain; scintillation and other simultaneous signal power impairments can also be considered but at the expense of a much more complicated and involved exposition. Such cases will be deferred to future publications.

II. A BRIEF OVERVIEW OF THE DYNAMICS PORTION OF THE ACTS RAIN ATTENUATION PREDICTION MODEL.

From the development of the ACTS Rain Attenuation Prediction Model (in particular, Section 5 of [3]), one models the temporal evolution of the link attenuation $A(t)$ through the parameter $x_A(t)$, i.e.,

$$x_A(t) = \frac{\ln A(t) - \ln A_m}{\sigma_{\ln A}} \quad (1)$$

where A_m is the median of the link attenuation and $\sigma_{\ln A}$ is the standard deviation of the logarithm of attenuation; these two parameters are specific to the particular location, frequency of operation, and geometry of the satellite link. It is the $x_A(t)$ parameter that is given by, in the most general case, a multi-component Markov random process which is determined by a system of first order stochastic differential equations. It is, however, expedient to reduce to a one component model with careful consideration given to the incorporation of the temporal smoothing process induced by the extended propagation path. In particular, as was done in [3], one can consider (approximately) the one component process given by the single first-order differential equation

$$\frac{dx_A}{dt} = -\gamma_S x_A + \sqrt{2\gamma_S} \xi(t) \quad (2)$$

where the random function $\xi(t)$ is governed by Gaussian statistics with a zero mean (i.e., "white noise"):

$$\langle \xi(t) \rangle = 0, \quad \langle \xi(t) \xi(t + \tau) \rangle = \delta(\tau)$$

and where γ_S is a "smoothed" temporal parameter that is given by the solution of the transcendental equation

$$\exp\left(-\frac{\gamma_1}{\gamma_S}\right) + \exp\left(-\frac{\gamma_2}{\gamma_S}\right) = \exp(-1) \quad (3)$$

From geometrical considerations concerning the propagation path, characteristic rain cell size, and long term spatial isotropy of rain cell movement, one has the following relation that exists between the coefficients γ_1 and γ_2 that appear in Eq.(3):

$$\gamma_2 = \gamma_1 \left(1 + \frac{2L \cos \theta}{\pi R_C}\right)^{-1/2} \quad (4)$$

where L is the total propagation path length within the potential rain region, θ is the link elevation angle, and $R_C = 4$ Km is the characteristic rain cell size as required by fundamental considerations and model constraints [3]. By the same considerations, one has for the coefficient γ_1

$$\gamma_1 = \frac{2v}{\pi R_C} \approx 0.1336 \text{ min}^{-1} \quad (5)$$

where $v = 14$ m/s is the characteristic speed of the rain cell that is assumed in the model. For a typical communications link with an elevation angle $\sim 40^\circ$, $\gamma_S \sim 0.06 \text{ min}^{-1} = 0.001 \text{ sec}^{-1}$.

In what follows, the one component Markov process $x_A(t)$ given by Eq.(2), with contact made to the link attenuation $A(t)$ through Eq.(1), will be used to derive an optimal fade detection and prediction algorithm for use with measured (real time) fade data.

III. DEVELOPMENT OF AN OPTIMAL FADE IDENTIFICATION AND PREDICTION ALGORITHM

A. The Observation and Sampling Model

Here, the problems of fade identification and prediction as defined in the introduction will be solved. At the outset, it is useful to consider the measurement process via the following *observation model* :

$$A_{obs}(t) = A(t) + n(t) \quad (6)$$

where $A(t)$ is the actual attenuation that exists on the communications link at time t , $n(t)$ is the associated measurement uncertainty or "noise", and $A_{obs}(t)$ is the observed link attenuation. The measurement noise, which can result from inaccuracies in the hardware of the measurement process or from an assumed measurement function such as that used in "frequency scaling" whereby a signal fade at one frequency is derived from a fade measurement at another frequency, is a random function of time and can also be statistically characterized by a white noise process, viz,

$$\begin{aligned} \langle n(t) \rangle &= 0 \\ \langle n(t_1)n(t_2) \rangle &= \sigma_n^2 \delta(t_1 - t_2) \end{aligned} \quad (7)$$

where σ_n is the standard deviation of the measurement noise. Remembering Eq.(1), one has from Eq.(6),

$$A_{obs}(t) = A(x_A, t) + n(t) , \quad A(x_A, t) \equiv A_m \exp(\sigma_{lnA} x_A(t)) \quad (8)$$

for the observation model where the random process x_A is governed by the stochastic differential equation Eq.(2). The following problem can now be defined: given the measured fade $A_{obs}(t)$ and given Eqs.(8), (7), (2), and (1), it is desired to obtain an estimate of the quantity $x_A(t)$ at time t (i.e., identification) as well as the extrapolated estimate of the quantity $x_A(t + \tau)$ that should prevail at a future time $t + \tau$ (i.e., prediction). Using Eq.(1), one can then easily obtain the corresponding values $A(t)$ and $A(t + \tau)$.

This is formally a problem in the optimal (with respect to a given criterion) estimation (or filtering) and prediction of a non-linear Markov continuous random process sampled in time.

Since the random process in question is the quantity x_A and it is desired to obtain information concerning the instantaneous value of this quantity from the measurement of A_{obs} , one must invariably deal with a parameter known as the *a posteriori* probability density $p(x_A(t) | A_{obs}(t))$ governing the random process at a time t conditioned on the observation of the quantity $A_{obs}(t)$. Once this probability density has been secured, one can easily obtain descriptions of the random process x_A in the form of statistical moments such as the average $x_A^*(t)$ (referred to as the *optimal estimate* minimized with respect to the mean-square error), the associated standard deviation $\sigma_{x_A}^*(t)$ of the optimal estimate, etc.

Although both $x(t)$ and $A_{obs}(t)$ are, in general, continuous in the time variable, the latter is actually sampled at discrete times t_i with a corresponding time interval T separating the samples, i.e., $T = t_i - t_{i-1}$. Thus, due to this sampling process overlaid on the continuous process $A_{obs}(t)$, one must also now consider additional random processes which result from the sampling, viz,

$$\tilde{A}_{obs}(t_i) = A_{obs}(t) \delta(t - t_i) \quad \text{and} \quad \tilde{x}_A(t_i) = x_A(t) \delta(t - t_i). \quad (9)$$

One must thus amend the *a posteriori* probability density $p(x_A(t) | A_{obs}(t))$ to reflect this circumstance and consider $p(x_A(t), \tilde{x}_A(t) | A_{obs}(t))$, viz, the probability density gov-

erning the continuous random process $x_A(t)$ and the sampled process $\tilde{x}_A(t)$ conditioned on the value of the discrete measurement of $A_{obs}(t)$.

B. The Statistical Identification of The Prevailing Link Attenuation Level

Now the random process $x_A(t)$, described by Eq.(2), has associated with it a transition (and conditional) probability density $p(x_A(t) | \tilde{x}_A(t_i))$ giving the statistics connected with the evolution of the sampled value $\tilde{x}_A(t_i)$ at a time t_i to the value of its continuous counterpart $x_A(t)$ a later time $t > t_i$. Following the results in [2,3] (in particular, Eqs.(53) and (54) of [3]), this transition probability density is given by the Kolmogorov equation associated with the first order differential equation Eq.(2), i.e.,

$$\frac{\partial p(x_A(t) | \tilde{x}_A(t_i))}{\partial t} = D_{x_A}[p(x_A(t) | \tilde{x}_A(t_i))], \quad t > t_i \quad (10)$$

where the differential operator $D_{x_A}[f(x_A)]$ for the Kolmogorov equation is given by

$$D_{x_A}[f(x_A)] \equiv \gamma_S \frac{\partial}{\partial x_A} (x_A f(x_A)) + \gamma_S \frac{\partial^2 f(x_A)}{\partial x_A^2} \quad (11)$$

In an analogous fashion, the probability density $p(x_A(t), \tilde{x}_A(t) | A_{obs}(t))$ is a transition probability density governing the random processes $x_A(t)$ and $\tilde{x}_A(t)$ conditioned on the set of measured observations $A_{obs}(t)$, each of which is connected with the random process in question via the observation model of Eq.(6) and the sampling model of Eq.(9). In this case, the Kolmogorov Equation of Eq.(10) must be augmented with this (non-linear) observation model. In particular, amending the observation model of Eq.(6) with the sampling model of Eq(9), one has that the quantity

$$\tilde{n}(t) = \tilde{A}_{obs}(t) - A(\tilde{x}_A, t) \quad (12)$$

is a zero mean Gaussian random variable with variance σ_n^2 (by Eq.(7)). As shown in the Appendix, using this fact in an extrapolated *a posteriori* probabilistic analysis for the evolution of $x_A(t)$, one obtains an integrodifferential equation for $p(x_A(t), \tilde{x}_A(t) | A_{obs}(t))$ a modified version of the Stratonovich Equation [4], well known in Markov filtering theory, viz,

$$\begin{aligned} \frac{\partial p(x_A(t), \tilde{x}_A(t) | A_{obs}(t_i))}{\partial t} = & D_{x_A} \left[p(x_A(t), \tilde{x}_A(t) | A_{obs}(t_i)) \right] + \left[\phi(\tilde{x}_A(t), t_i) - \right. \\ & \left. - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi(\tilde{x}'_A(t), t_i) p(x'_A(t), \tilde{x}'_A(t) | A_{obs}(t_i)) d\tilde{x}'_A dx'_A \right] \times \\ & \times p(x_A(t), \tilde{x}_A(t) | A_{obs}(t_i)) \quad (13) \end{aligned}$$

where

$$\phi(\tilde{x}_A(t), t_i) \equiv -\frac{1}{2\sigma_n^2} (A_{obs}(t_i) - A(\tilde{x}_A, t_i))^2 \quad (14)$$

It is important to note that Eq.(13) holds for times t within the interval $t_i < t < t_{i+1}$.

Obtaining a solution from Eq.(13) commences with writing the *a posteriori* probability density in the form

$$p(x_A(t), \tilde{x}_A(t) | A_{obs}(t_i)) = p(x_A(t) | \tilde{x}_A(t)) p(\tilde{x}_A(t) | A_{obs}(t_i)) \quad (15)$$

Substituting Eq.(15) into Eq.(13), integrating the result with respect to x , and using the facts that

$$\int_{-\infty}^{\infty} p(x_A(t) | \tilde{x}_A(t)) dx_A = 1 \quad \text{and} \quad \int_{-\infty}^{\infty} D_{x_A} [p(x_A(t) | \tilde{x}_A(t))] dx_A = 0$$

(such integrals indicate integration over all the possible values realized by x_A at a fixed time t and where the latter relation is obtained using Eq.(11) and the boundary conditions that $p(x_A(t) | \tilde{x}_A(t))$ and its first derivative vanish at $x = \pm \infty$) yields the relation

$$\frac{\partial p(\tilde{x}_A(t) | A_{obs}(t_i))}{\partial t} = \left[\phi(\tilde{x}_A(t), t_i) - \int_{-\infty}^{\infty} \phi(\tilde{x}_A'(t), t_i) p(\tilde{x}_A'(t) | A_{obs}(t_i)) d\tilde{x}_A' \right] p(\tilde{x}_A(t) | A_{obs}(t_i)) \quad (16)$$

for $t_i < t < t_{i+1}$. By substituting Eq.(16) back into Eq.(13), one then re-derives Eq.(10) prescribing the probability density $p(x_A(t) | \tilde{x}_A(t))$ thus making the proposed solution of Eq.(15) self-consistent.

An approximate analytic expression can be obtained for Eq.(16). One notes that the second term within the brackets is simply the average $\langle \Phi(t) \rangle$ over all possible values of \tilde{x}_A . Since it is only a function of t , and if the time interval $t - t_i$ is small as compared to the characteristic time of variation for this average, one can neglect it and obtain a simpler form which has the solution

$$p(\tilde{x}_A(t) | A_{obs}(t_i)) = N \exp \left[\int_{t_i}^t \phi(\tilde{x}_A(t'), t_i) dt' \right] p(\tilde{x}_A(t_i) | A_{obs}(t_i)) \quad (17)$$

where the normalization constant N is given by

$$N = \left(\int_{-\infty}^{\infty} \exp \left[\int_{t_i}^t \phi(\tilde{x}_A(t'), t_i) dt' \right] p(\tilde{x}_A(t_i) | A_{obs}(t_i)) \right)^{-1} \quad (18)$$

Equations (17) and (18) define the "partition" representation of the *a posteriori* probability density function [5].

One now admits the well established Gaussian approximation, which demands that the probability density $p(\tilde{x}_A(t_i) | A_{obs}(t_i))$ take the form

$$p(\tilde{x}_A(t_i) | A_{obs}(t_i)) = \sqrt{\frac{1}{2\pi K(t_i)}} \exp\left(-\frac{(\tilde{x}_A(t_i) - \hat{x}_A(t_i))^2}{2K(t_i)}\right) \quad (19)$$

where $\hat{x}_A(t_i)$ is the extrapolated estimate of the random process $x_A(t_i)$ available at time t_i and $K(t_i)$ is the standard deviation of this estimate about the actual value; the extrapolated estimate, which is obtained from the optimal estimate of the previous time interval, is employed here since it is the only *a posteriori* information that one has at time t_i about the random process $x_A(t)$ to combine with the observed quantity $A_{obs}(t_i)$ to yield the statistics that govern the possible values for $\tilde{x}_A(t_i)$. For this reason, $K(t)$ is sometimes referred to as *the standard deviation (or, in general, the covariance) associated with the errors of the extrapolated estimate*. To make the problem defined by the substitution of Eq.(19) into Eqs.(17) and (18) analytically amenable, one expands the function $\phi(\tilde{x}_A(t_i), t_i)$ that appears within the exponential functions in Eqs.(17) and (18) into a Taylor series about the optimal estimate $\hat{x}_A(t)$ obtained at the i^{th} sampling time t_i , retaining terms up to the second to match that of Eq.(19). Using Eq.(9) in Eq.(14) and substituting this composite result into the expansion and using Eq.(19), one obtains from Eqs.(17) and (18)

$$p(\tilde{x}_A(t) | A_{obs}(t_i)) = \frac{\exp\left(a(t_i)(\tilde{x}_A - \hat{x}) - \frac{(\tilde{x}_A - \hat{x})^2}{2\sigma_{x_A}^*(t_i)}\right)}{\sqrt{2\pi\sigma_{x_A}^*(t_i)} \exp\left(\frac{a^2(t_i)\sigma_{x_A}^*(t_i)}{2}\right)} \quad (20)$$

where the quantity $\sigma_{x_A}^*(t_i)$ is the error covariance of the optimal estimate (the latter will be defined shortly) given by

$$\sigma_{x_A}^*(t_i) \equiv \left(K^{-1}(t_i) - b(t_i) \right)^{-1} \quad (21)$$

and

$$a(t_i) \equiv \left. \frac{\partial \phi_s(\tilde{x}, t_i)}{\partial \tilde{x}} \right|_{\tilde{x} = \hat{x}}, \quad b(t_i) \equiv \left. \frac{1}{2} \frac{\partial^2 \phi_s(\tilde{x}, t_i)}{\partial \tilde{x}^2} \right|_{\tilde{x} = \hat{x}} \quad (22)$$

are coefficients that contain the observed sampled values $A_{obs}(t_i)$; here, $\phi_s(\tilde{x}_A(t_i), t_i) = \phi(\tilde{x}_A(t), t) \delta(t - t_i)$. Using Eq.(14), one then obtains

$$a(t_i) = \left(\frac{\sigma_{lnA}}{\sigma_n^2} \right) A(\hat{x}, t_i) (A_{obs}(t_i) - A(\hat{x}_A, t_i)) \quad (23)$$

$$b(t_i) = \left(\frac{2}{\sigma_n^2} \right) A(\hat{x}, t_i) (A_{obs}(t_i) - 2A(\hat{x}_A, t_i))$$

Using the *a posteriori* probability density of Eq.(20), one can now obtain an expression for the optimal estimate $x^*(t_i)$ of the value of $\tilde{x}_A(t_i)$ connected with the observational data $A_{obs}(t_i)$. Of the many optimality criteria that can be used to define a particular estimate, the one selected here is the simplest, viz, the criterion that minimizes the mean square error which yields the relation

$$x_A^*(t_i) = \int_{-\infty}^{\infty} \tilde{x}_A(t_i) p(\tilde{x}_A(t_i) | A_{obs}(t_i)) d\tilde{x}$$

Substituting Eq.(20) into this and performing the integration yields the following recursion relationship for the optimal estimate:

$$x_A^*(t_i) = \hat{x}_A(t_i) + \sigma_{x_A}^*(t_i) a(t_i) \quad (24)$$

As mentioned earlier, the extrapolated estimate $\hat{x}_A(t_i)$ is that obtained from the optimal estimate of the previous sampling time t_{i-1} through the use of the transition probability density defined by Eq.(10). The solution of this equation is well known [2,3] and is given by

$$p(x_A(t_i) | \tilde{x}_A(t_{i-1})) = \sqrt{\frac{1}{2\pi(1-\Phi_i^2)}} \exp\left[-\frac{(x_A(t_i) - \Phi_i^2 \tilde{x}_A(t_{i-1}))^2}{2(1-\Phi_i^2)}\right] \quad (25)$$

where $\Phi_i \equiv \exp(-\gamma_S(t_i - t_{i-1}))$. The extrapolated estimate $\hat{x}_A(t_i)$ at time t_i given the *a posteriori* value $\tilde{x}_A(t_{i-1}) = x_A^*(t_{i-1})$ is simply

$$\hat{x}_A(t_i) = \int_{-\infty}^{\infty} x_A p(x_A(t_i) | \tilde{x}_A(t_{i-1})) dx_A$$

Upon substituting Eq.(25) into this relationship and integrating, one obtains

$$\hat{x}_A(t_i) = \Phi_i x_A^*(t_{i-1}) \quad (26)$$

which, when used in Eq.(24), demonstrates the recursive nature of the estimate.

It now remains to obtain a similar expression for the standard deviation of the extrapolated estimate $K(t)$. Equation (19) essentially provides a working definition for this quantity. In particular, one has

$$K(t_i) = \int_{-\infty}^{\infty} \left(\tilde{x}_A(t_i) - \hat{x}_A(t_i) \right)^2 p(\tilde{x}_A(t_i) | A_{obs}(t_i)) d\tilde{x}$$

In order to relate a value of this parameter at t_i to the *a posteriori* values obtained at time t_{i-1} , one can use Eq.(15) to write the relationship

$$K(t_i) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(\tilde{x}_A(t_i) - \hat{x}_A(t_i) \right)^2 p(\tilde{x}_A(t_i), \tilde{x}_A(t_{i-1}) | A_{obs}(t_{i-1})) d\tilde{x} d\tilde{x}' \quad (27)$$

Using the previously derived expressions of Eqs.(19) and (25) in Eq.(27) yields the recursive relationship

$$K(t_i) = 1 + \Phi_i^2 \left(\sigma_{x_A}^*(t_{i-1}) - 1 \right) \quad (28)$$

that relies on former values of $\sigma_{x_A}^*(t_i)$ obtained from Eq.(21).

Equations (24), (21), (26), and (28), in addition to the auxiliary relations of Eq.(23) collectively compose the dynamic identification part of the problem defined at the outset. The approximation used that allows one to write Eq.(17) as well as considerations of the errors incurred in the optimal estimate place upper limits on the time interval $[t_{i-1}, t_i]$ between consecutive samples. However, the typical clock intervals ~ 1 second that are encountered in practice are easily within this limit. It now remains to obtain a prescription for the dynamic prediction of link fade levels into the future based on the data afforded from this fade identification process.

C. The Prediction of Link Attenuation for Short Times into the Future

From some results of the foregoing, in particular, the solution given by Eq.(25) for the transition probability density of the attenuation process, one can derive a relation yielding the average value of the link attenuation at a time T_{pred} into the future. In particular, the problem can be succinctly defined as follows: Given the fact that an optimal estimate $x_A^*(t_i)$

is obtained for the attenuation process at the time t_i , and *on the hypothesis that the attenuation will increase*, it is of interest to obtain the extrapolated estimate $\widehat{\Delta x}_A(t_i + T_{pred})$ of the change in the attenuation that will occur at a later time $t_i + T_{pred}$. One then has for the total value of this combined extrapolated optimal estimate of the attenuation process

$$\widehat{x}_A^*(t_i + T_{pred}) = x_A^*(t_i) + \widehat{\Delta x}_A(t_i + T_{pred}) \quad (29)$$

By this formulation of the problem, one has

$$\begin{aligned} \widehat{\Delta x}_A(t_i + T_{pred}) &= \\ &= \int_0^{\infty} \Delta x_A(t_i + T_{pred}) p \left(x_A(t_i + T_{pred}) > x_A(t_i) + \Delta x_A(t_i + T_{pred}) \mid x_A^*(t_i) \right) d(\Delta x_A) \end{aligned} \quad (30)$$

where the conditional probability density is given by

$$\begin{aligned} p \left(x_A(t_i + T_{pred}) > x_A(t_i) + \Delta x_A(t_i + T_{pred}) \mid x_A^*(t_i) \right) &\equiv \\ &\equiv \int_{x_A(t_i) + \Delta x_A(t_i + T_{pred})}^{\infty} p(x_A'(t) \mid x_A^*(t_i)) dx_A' \end{aligned} \quad (31)$$

To this end, substituting Eq.(25) into Eq.(31) and this result into Eq.(30) yields, upon a change of variables,

$$\widehat{\Delta x}_A(t_i + T_{pred}) = \frac{1}{2} \left[\int_{x_A^*(1-\Phi_{pred})}^{\infty} \operatorname{erfc} \left(\frac{y}{\sqrt{2D(T_{pred})}} \right) y dy - \right.$$

$$\left. - x_A^* (1 - \Phi_{pred}) \int_{x_A^* (1 - \Phi_{pred})}^{\infty} \operatorname{erfc} \left(\frac{y}{\sqrt{2D(T_{pred})}} \right) dy \right] \quad (32)$$

where $\Phi_{pred} \equiv \exp(-\gamma_S T_{pred})$ and $D(T_{pred}) \equiv 1 - \Phi_{pred}^2$. In the approximation in which one can assume that $x_A^* (1 - \Phi_{pred}) \approx 0$ relative to the range of integration and the contributions of the integrands, the integrals in Eq.(32) can be analytically evaluated and, in conjunction with Eq.(29), gives for the extrapolated optimal estimate of attenuation for a future time $t_i + T_{pred}$ based on the estimate at time t_i

$$\widehat{x}_A^*(t_i + T_{pred}) = \Psi_{pred} x_A^*(t_i) + \frac{D(T_{pred})}{4} \quad (33)$$

where

$$\Psi_{pred} \equiv 1 - \sqrt{\frac{D(T_{pred})}{2\pi}} (1 - \Phi_{pred}) \quad (34)$$

A limitation is placed upon the maximum value of T_{pred} by the approximation used to evaluate the integrals of Eq.(32); in particular, one has

$$T_{pred} \ll \frac{\sigma_{\ln A}}{2\gamma_S (\ln A(t) - \ln A_m)}$$

showing that, as can be expected, the value is a function of the prevailing link attenuation $A(t)$. A more rigorous analysis for this maximum value would involve considering the prediction error associated with $\widehat{\Delta x}_A(t_i + T_{pred})$. However, since in the applications, one usually only needs to predict no more than two to four time intervals ahead (i.e., typically ~ 2 to 4 seconds), this condition is easily met.

IV. ALGORITHMIC IMPLEMENTATION OF THE FOREGOING

The synthesis of the relevant relations derived above is shown in Fig.1 and comprises two separate sections: one that performs fade identification and the other that performs fade prediction. In the identification section, **D1** and **D2** perform as discriminators carrying out the operations $\left[A_{obs}(t) - A(\hat{x}_A(t_i)) \right]$ and $\left[A_{obs}(t) - 2A(\hat{x}_A(t_i)) \right]$, respectively, multipliers **M1** and **M2** form the operations needed to complete the emulation of the relations given in Eq.(23), the inverters **I** perform multiplicative inversion, and **S** denotes the emulation of the operation yielding $A_m \exp(\sigma_{lnA} x_A(t))$. Also, Φ_i denotes the multiplication by the discrete time transition factor $\exp(-\gamma_S T_i)$, Φ_i^2 is the square of this factor, and T_i is a time delay of T_i seconds. In the prediction section, once a time interval T_{pred} is selected over which a fade prediction is to be made, **D** (T_{pred}) evaluates the operation $1 - \exp(-2\gamma_S T_{pred})$, Ψ_{pred} then performs the operation as specified by Eq.(34), and finally multiplier **M3** forms the second member of the right side of Eq.(33).

Such an algorithm can be implemented either in software or hardware and is easily included into the operation of any satellite earth station. Although the *structure* of the algorithm remains the same in every application, several of the numerical coefficients which it employs reflects the specific geometry and operating frequency of a satellite link as well as the location of the earth terminal. For example, γ_S is a function of the propagation path length through the potential rain region as well as the elevation angle, as shown by Eq.(4). Not only is the link operating frequency reflected in the selection of the parameters A_m and σ_{lnA} that appear in each **S** module of Fig.1, but also the specific location of the earth station due to the extremely detailed rain statistics data base of the ACTS rain attenuation model (a geographical resolution of $\sim 0.5^\circ$ in latitude and longitude in the continental US, essentially equivalent to about 6000 rain "zones") from which they are calculated. The software implementation of the ACTS Rain Attenuation Prediction Model for this and other general satellite system link design is available from COSMIC, NASA Software for Industry, The University of Georgia, Athens, GA 30602, U.S.A.

APPENDIX - A Derivation of Equation 13

Here, it is desired to augment the dynamic description of the random process $x_A(t)$ that is afforded by the Kolmogorov equation, Eq. (10), with observed *a priori* measurements $A_{obs}(t_i)$ of the link attenuation which, through Eq.(8), is a convolution of $\tilde{x}_A(t_i)$ as well as the measurement noise $n(t)$. It is thus expedient to consider a quantity that incorporates both aspects of *a priori* observation and dynamical transition, i.e., the conditional transition probability density $p\left(x_A(t+\Delta t), \tilde{x}_A(t+\Delta t), A_{obs}(t_i+\Delta t) | A_{obs}(t_i)\right)$ which incorporates two successive observations at time t_i and at time $t_i+\Delta t$ with $\Delta t = t_{i+1} - t_i$ used to explicitly denote the prevailing time increment, and with the proviso that $t_i \leq t < t_{i+1}$. This probability density can be decomposed into two equivalent cascaded forms

$$p\left(A_{obs}(t_i + \Delta t) | A_{obs}(t_i), x_A(t + \Delta t), \tilde{x}_A(t + \Delta t)\right) \times \\ \times p\left(x_A(t + \Delta t), \tilde{x}_A(t + \Delta t) | A_{obs}(t_i)\right)$$

and

$$p\left(x_A(t + \Delta t), \tilde{x}_A(t + \Delta t) | A_{obs}(t_i), A_{obs}(t_i + \Delta t)\right) p\left(A_{obs}(t_i + \Delta t) | A_{obs}(t_i)\right)$$

Equating these two representations and rearranging factors gives

$$p\left(x_A(t + \Delta t), \tilde{x}_A(t + \Delta t) | A_{obs}(t_i), A_{obs}(t_i + \Delta t)\right) = F(\Delta t) \times \\ \times p\left(A_{obs}(t_i + \Delta t) | A_{obs}(t_i), x_A(t + \Delta t), \tilde{x}_A(t + \Delta t)\right) \times \\ \times p\left(x_A(t + \Delta t), \tilde{x}_A(t + \Delta t) | A_{obs}(t_i)\right) \quad (A1)$$

where $F(\Delta t) \equiv \left[p\left(A_{obs}(t_i + \Delta t) | A_{obs}(t_i)\right)\right]^{-1}$ is a factor that is independent of the process x_A . The first probability density function on the right side of Eq.(A1) is connected with the

uncertainty associated with the measurement process. Noting that the measurement noise at $t_i + \Delta t$ only affects the random quantity $A_{obs}(t_i + \Delta t)$ and since it is only the sampled value $\tilde{x}_A(t + \Delta t)$ that is connected with this sampled Gaussian noise process as defined by Eq.(7), one has the following development:

$$\begin{aligned}
p\left(A_{obs}(t_i + \Delta t) | A_{obs}(t_i), x_A(t + \Delta t), \tilde{x}_A(t + \Delta t)\right) &= p\left(A_{obs}(t_i + \Delta t) | \tilde{x}_A(t + \Delta t)\right) \\
&= N \exp\left(-\frac{n^2(t_i + \Delta t)}{2\sigma_n^2} \Delta t\right) = N \exp\left[\phi\left(\tilde{x}_A(t + \Delta t), t + \Delta t\right) \Delta t\right] \\
&\approx N \left[1 + \phi\left(\tilde{x}_A(t + \Delta t), t + \Delta t\right) \Delta t + \dots\right] \tag{A2}
\end{aligned}$$

where N is a normalization constant and use was made of Eqs.(12) and (14) and the fact that Δt is sufficiently small to warrant the series expansion in the last line. The second probability density in Eq.(A1) is an extrapolated probability density function of the process x_A based on the *a priori* observation $A_{obs}(t_i)$. Expanding this quantity into the first two terms of its series representation yields

$$\begin{aligned}
p\left(x_A(t + \Delta t), \tilde{x}_A(t + \Delta t) | A_{obs}(t_i)\right) &\approx p\left(x_A(t), \tilde{x}_A(t) | A_{obs}(t_i)\right) + \\
&+ \frac{dp\left(x_A(t), \tilde{x}_A(t) | A_{obs}(t_i)\right)}{dt} \Delta t \tag{A3}
\end{aligned}$$

After the initial *a priori* input $A_{obs}(t_i)$ at time t_i , no other such information is used during the time interval Δt thus allowing the time derivative in to be described by the Kolmogorov equation, Eq.(10). Making this substitution and inserting this result and that of Eq.(A2) into Eq.(A1) and retaining terms up to first order in Δt yields

$$p\left(x_A(t + \Delta t), \tilde{x}_A(t + \Delta t) | A_{obs}(t_i), A_{obs}(t_i + \Delta t)\right) = C \left[p\left(x_A(t), \tilde{x}_A(t) | A_{obs}(t_i)\right) + \right.$$

$$\begin{aligned}
& +D_{x_A} \left[p \left(x_A(t), \tilde{x}_A(t) \mid A_{obs}(t_i) \right) \right] \Delta t + \\
& \left. + \phi \left(\tilde{x}_A(t+\Delta t), (t+\Delta t) \right) p \left(x_A(t), \tilde{x}_A(t) \mid A_{obs}(t_i) \right) \Delta t \right] \quad (A4)
\end{aligned}$$

where $C = F(\Delta t)N$ is a constant (at least for a fixed Δt) which must now be evaluated. To this end, one integrates both sides of this equation over all values of $x_A(t)$ and $\tilde{x}_A(t)$ and, noting the analogous relationships that follow Eq.(15), obtains

$$\begin{aligned}
C &= \left[1 + \Delta t \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi \left(\tilde{x}_A(t+\Delta t), (t+\Delta t) \right) p \left(x_A(t), \tilde{x}_A(t) \mid A_{obs}(t_i) \right) dx_A d\tilde{x}_A \right]^{-1} \\
&\approx 1 - \Delta t \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi \left(\tilde{x}_A(t+\Delta t), (t+\Delta t) \right) p \left(x_A(t), \tilde{x}_A(t) \mid A_{obs}(t_i) \right) dx_A d\tilde{x}_A \quad (A5)
\end{aligned}$$

where, once again, the fact was used whereby Δt is small. Substituting this result back into Eq.(A4) and retaining terms up to first order in Δt yields

$$\begin{aligned}
& p \left(x_A(t+\Delta t), \tilde{x}_A(t+\Delta t) \mid A_{obs}(t_i), A_{obs}(t_i + \Delta t) \right) = \\
& = p \left(x_A(t), \tilde{x}_A(t) \mid A_{obs}(t_i) \right) + D_{x_A} \left[p \left(x_A(t), \tilde{x}_A(t) \mid A_{obs}(t_i) \right) \right] \Delta t + \\
& + \left[\phi \left(\tilde{x}_A(t+\Delta t), (t+\Delta t) \right) - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi \left(\tilde{x}_A'(t+\Delta t), t+\Delta t \right) \times \right. \\
& \left. \times p \left(x_A'(t), \tilde{x}_A'(t) \mid A_{obs}(t_i) \right) dx_A' d\tilde{x}_A' \right] p \left(x_A(t), \tilde{x}_A(t) \mid A_{obs}(t_i) \right) \Delta t
\end{aligned}$$

Finally, moving the first member on the right side to the left side and dividing through by Δt forms a different quotient which reduces to Eq.(13) after taking the limit $\Delta t \rightarrow 0$.

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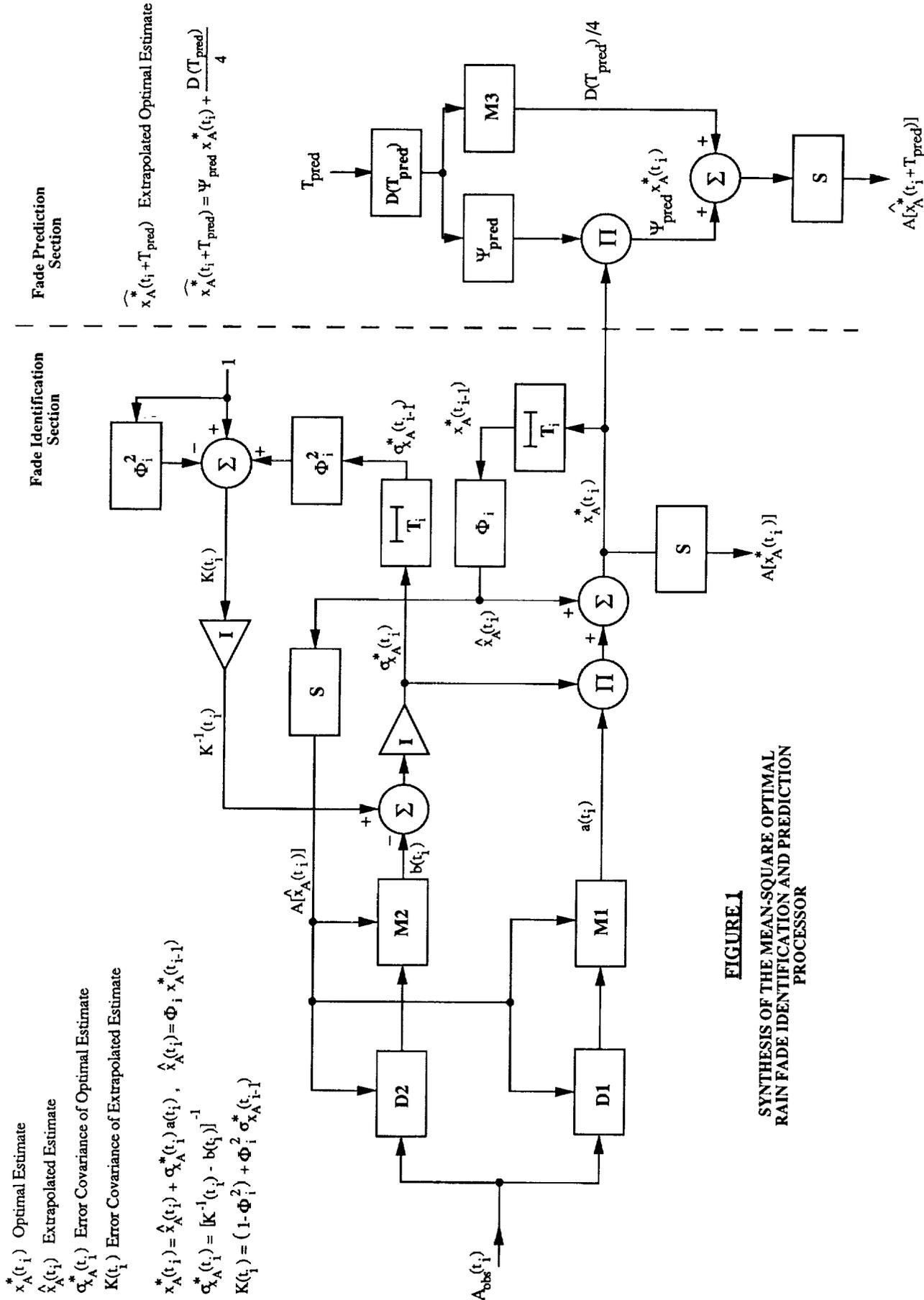


FIGURE 1
SYNTHESIS OF THE MEAN-SQUARE OPTIMAL
RAIN FADE IDENTIFICATION AND PREDICTION
PROCESSOR



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16. Abstract The dynamic and composite nature of propagation impairments that are incurred on earth-space communications links at frequencies in and above the 30/20 GHz Ka band, i.e., rain attenuation, cloud and/or clear air scintillation, etc., combined with the need to counter such degradations after the small link margins (typical of such frequency bands) have been exceeded, necessitate the use of dynamic statistical identification and prediction processing of the fading signal in order to optimally estimate and predict the levels of each of the deleterious attenuation components. Such requirements are being met in NASA's Advanced Communications Technology Satellite (ACTS) Project by the implementation of optimal processing schemes derived through the use of the ACTS Rain Attenuation Prediction Model and nonlinear Markov filtering theory. The ACTS Rain Attenuation Prediction Model discerns climatological variations on the order of 0.5° in latitude and longitude in the continental U.S. The static portion of the model gives precise availability predictions for the "spot beam" links of ACTS. However, the structure of the dynamic portion of the model, which yields performance parameters such as fade duration probabilities, etc., is isomorphic to the state-variable approach of stochastic control theory and, as shown in this paper, is amenable to the design of such statistical fade processing schemes which can be made specific to the particular climatological location at which they are employed.					
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